IISEE Lecture Note
2011

FOUNDATION ENGINEERING 2

By

Songtao XUE

Department of Architecture, Faculty of Engineering
Tohoku Institute of Technology

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International Institute of Seismology and Earthquake Engineering
(IISEE)
Building Research Institute

Syllabus

Subject: Foundation Engineering 2

Lecturer: Songtao XUE

Day: 1

Contents:

A foundation is the part of an engineered structure that transmits the structure's forces into the soil and rock that supports it. The shape, depth, and materials of the foundation design depend on the many factors including the structural loads, the existing ground conditions, and local material availability.

Proper design of building foundation requires the knowledge of (a) external loads and loads transmitted by the building superstructure, (b) local code requirements, (c) nature and composition of different types of soil at the site, (d) behavior and stress-related deformability of soils supporting the foundation system, and (e) general geological conditions of the site. Together with knowledge of such scientific principles, rational engineering judgment acquired through observation and experience is indispensable in the foundation engineering practice.

It is evident that this one-day lecture cannot attempt to cover all these aspects. The objective here is to cover some basic aspects of the design and construction of building foundations, including scientific principles as well as practical aspects. It is expected that the balance between theoretical and practical aspects in the content would provide the trainees with the clear overview of the essentials of building foundations.

Special Mentioning

The level of this lecture achieves the PE (Professional Engineer) level and the contents, the examples and practice problems are all in the same level with the PE (Professional Examination), but the units are different. We used SI unit system instead of the American system.

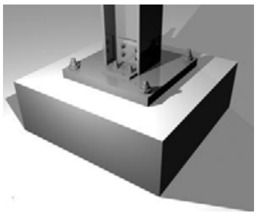
Contents:

Part 1. Shallow Foundations

Part 2. Pile and Deep Foundations

Part 3. Retaining Wall

Soil Mechanics and Foundation Engineering Lectures



Part 1. Shallow Foundations

Prof. Dr. Songtao XUE

Tohoku Institute of Technology

Tongji University

This lecture

Part 1.
Part 2.
Part 3.

- The level of this lecture achieves the level and the contents, the examples and practice problems are all in the same level with the but the units are different. We used SI unit system instead of the American system.

CONTENT

- **Introduction of Foundations**
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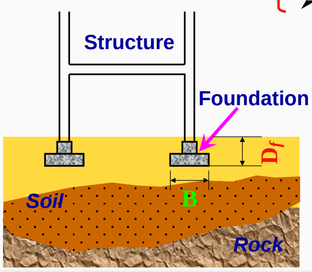
- **Bearing Capacity of Clay and Sand**
- **Effects of Water table on Footing Design**
- **Eccentric Loads on Rectangular Footings**
- **Rafts On Clay**
- **Examples and Problems**

Introduction of Foundations

- **Function** : to transfer the forces from the structure to the soil or rock without excessive settlement.
- **Foundation Types:**



- **Shallow Foundation**: the **depth** of the foundation is shallow relative to its **width**,

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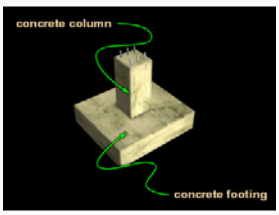
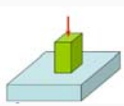
- **Category of Shallow Foundation**

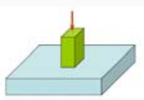
➢**Spread footings**



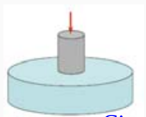
Spread footings



Square



Rectangular



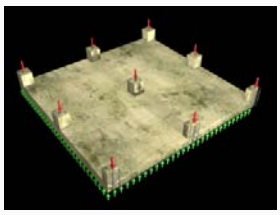
Circular

Other Shapes

- **Category of Shallow Foundation**

➢ **Continuous (or wall) footings**

➢ **Mats or rafts**



Mats or rafts

Main considerations in designing shallow foundation

Bearing Capacity failure and excessive Settlements

Bearing Capacity Failure

Excessive settlement

Bearing Capacity:

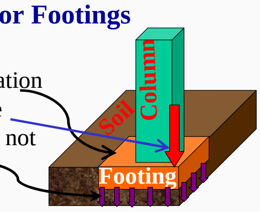
Settlements:

General Bearing Capacity

General Consideration for Footings

Footing and its Function:

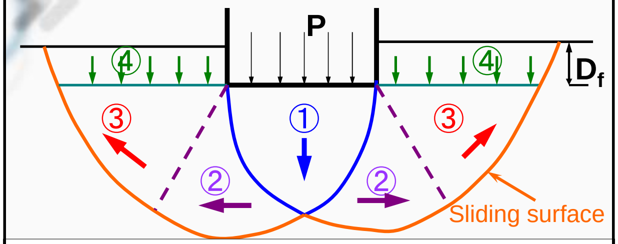
- Widened parts of the foundation
- Accepts **load** from structure
- Transmit **Forces** to the soil, not to exceed bearing capacity



General Considerations for design of footings:

- Located **below** the frost line and the moisture content change level.
- Be **safe against** overturning, sliding and uplift.
- **Satisfy** the allowable soil pressure
-

Soil deformation and bearing force



Allowable Bearing Capacity

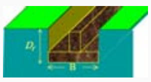
The ***allowable bearing capacity*** (Also: *net allowable bearing pressure* or *safe bearing pressure*) is the net pressure in excess of the overburden stress that will not cause shear failure or excessive settlements.

Typical Allowable Soil Bearing Capacities

Type of soil	Allowable pressure (kPa)
Massive Crystalline Bedrock	200
Sedimentary and foliated rock	100
Sandy gravel and/or gravel	100
Sand, silty sand, clayey sand, silty gravel and clayey gravel	
Clay, sandy clay, silty clay, and clayey silt	

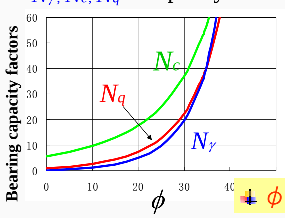
General Bearing Capacity Equation

- The **ultimate (or gross) bearing capacity** for a shallow **wall footing** is given as,



p_q : additional surface surcharge

N_γ, N_c, N_q : capacity factors.

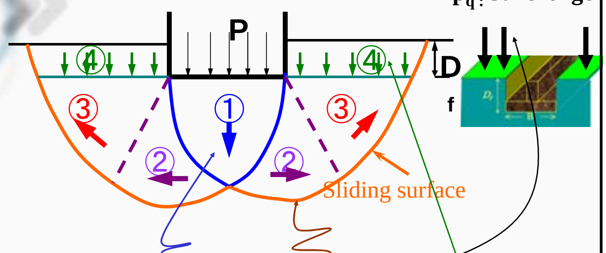


N_γ, N_c, N_q should be multiplied by factors for other shape and depth of footings:

- **Capacity factors**
- **Shape and Depth factors**
- ϕ : angle of internal friction

Introduction of the coefficients

p_q : Surcharge



$q_{ult} = \quad + \quad + (\quad + \quad N_q$

Bearing Capacity factors

Terzaghi

ϕ	N_c	N_q	N_γ
0.0	5.7	1.0	0.0
5.0	7.3	1.6	0.5
10.0	9.6	2.7	1.2
15.0	12.9	4.4	2.5
20.0	17.7	7.4	5.0
25.0	25.1	12.7	9.7
30.0	37.2	22.5	19.7
34.0	52.6	36.5	35.0
35.0	57.8	41.4	42.4
40.0	95.7	81.3	100.4
45.0	172.3	173.3	297.5
48.0	258.3	287.9	780.1
50.0	347.5	415.1	1153.2

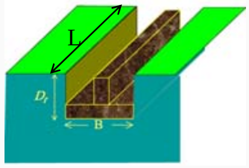
Meyerhof and Vesic

ϕ	N_c	N_q	N_γ	N_γ^b
0	5.14	1.00	0.00	0.00
5	6.50	1.60	0.07	0.50
10	8.30	2.50	0.37	1.20
15	11.00	3.90	1.10	2.60
20	14.80	6.40	2.90	5.40
25	20.70	10.70	6.80	10.80
30	30.10	18.40	15.70	22.40
32	35.50	23.20	22.00	30.20
34	42.20	29.40	31.20	41.10
36	50.60	37.70	44.40	56.30
38	61.40	48.90	64.10	78.00
40	75.30	64.20	93.70	109.40
42	93.70	85.40	139.30	155.60
44	118.40	155.30	211.40	224.60
46	152.10	158.50	328.70	330.40
48	199.30	222.30	526.50	496.00
50	266.90	319.10	873.90	762.90

Shape factors

N_c and N_γ multipliers for various values of **B/L**

B/L	N_c	N_γ
1 (square)	1.25	0.85
0.50	1.12	0.90
0.20	1.05	0.95
0.00	1.00	1.00
1 (circular)	1.20	0.70



Depth factor suggested to be applied for the N_c is

Depth factor for N_c:

K is a constant which is 0.2~0.4

- **Net and allowable Bearing Capacity**

Net Bearing Capacity is just the foundation weight taken away from the ultimate capacity.

Allowable Bearing Capacity

F **:safety factor** between ased on q_{net}).

Smaller value is sometimes used for transient load conditions such as from wind and seismic forces.

Bearing Capacity of Clay and Sand

- **Bearing Capacity of Clay**

Undrained case or $\phi = 0^\circ$ case:

Pore pressure

Undrained shear strength ← $Su = c$ ⇐ $\phi = 0^\circ$ — short time

cohesion

If no surface surcharge

$p_q = 0$ $N_\gamma = 0,\ N_q = 0$

Cohesion dominate bearing capacity

Bearing Capacity of Sand

For ideal sand , $c = 0$, then:

$$q_{ult} = \tfrac{1}{2}\rho g B N_\gamma + \left(p_q + \rho g D_f\right) N_q$$

If no surface surcharge:

$$q_{net} = q_{ult} - \rho g D_f = \tfrac{1}{2}\rho g B N_\gamma + \rho g D_f\left(N_q - 1\right)$$

Depth term $\rho g D_f N_q$ dominate bearing capacity

$$q_a = \frac{q_{net}}{F} = \frac{\rho g B}{F}\left(\tfrac{1}{2}\rho g N_\gamma + \rho g\left(N_q - 1\right)\left(\frac{D_f}{B}\right)\right)$$

depend on ϕ **governed by D_f/B**

Assuming: $F = 2,\ \rho g = 15.7\text{kN/m}^3$

[in kPa]

Bearing Capacity of Sand (continued)

Overburden (kPa)	C_n
0	2.00
24	1.45
48	1.21
96	1.00
144	0.87
192	0.77
240	0.70
287	0.63
335	0.58
383	0.54
431	0.50
479	0.46

Overburden load $\gamma \cdot D_f$ in Eq.

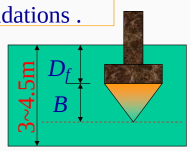
$$q_a = 10.53 C_n N$$

is assumed approximately 95.76kPa

Corresponding N-values is from 3~4.5m below the surface.

Correction factor needed for shallower foundations .

In practice, lowest average N-value from depth of $D_f + B$



N: standard penetration test value

Effects of Water table on Footing Design

•**General principle 1**: For cohesive $(\phi = 0^\circ)$ soils

Effects of Water table on Footing Design

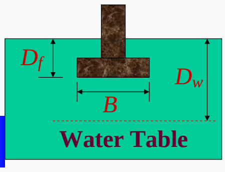
•**General principle 2**: For sand, use the submerged density $\rho_b = \rho_d - 1000\text{kg/m}^3$ in the equation for bearing capacity: $q_{ult} = \frac{1}{2}\rho g B N_\gamma + cN_c + (p_q + \rho g D_f)N_q$

The bearing capacity of a footing with the water table at the ground surface is half of the dry bearing capacity. **For accurate estimate:**

(a) **Water table is at the base of the footing: $D_f = D_w$**

For ideal sand , **c = 0:**

$$q_{ult} = \tfrac{1}{2}\rho_b g B N_\gamma + \rho_d g D_f N_q$$



General principle 2 (continue)

(b) **When water table is at the surface**: D_w=0

(c) **When water table is between the surface and the footing: $0 < D_w < D_f$**

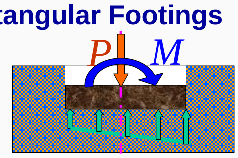
$$q_{ult} = \tfrac{1}{2}\rho_b g B N_\gamma + \left(p_q + \rho_d g D_w + \left(\rho_d - 1000\frac{kg}{m^3}\right) g\left(D_f - D_w\right)\right) N_q$$

Effects of Water table on Footing Design

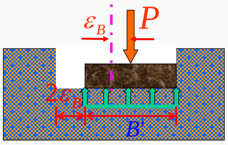
•**General principle 3**: For sand, if $D_w > D_f + B$ **the bearing capacity is not affected.**

Eccentric Loads on Rectangular Footings

•Use equivalent centric loads conditions for eccentric loads effect.



Equivalent Dimensions are:



$B' = B - 2\varepsilon_B;\ L' = L - 2\varepsilon_L$

$A' = B'L'$

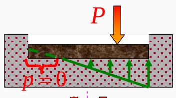
•**Use** for bearing capacity equation: $q_{ult} = \frac{1}{2}\rho g B' N_\gamma + cN_c + (p_q + \rho g D_f)N_q$

•**Use B'/L' to determine shape factors.**

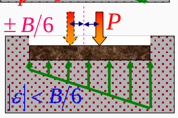
Eccentric Loads (continued)

Although the eccentricity is independent of the footing dimensions, a **trial-and-error solution** may be necessary when **designing footing**.

Assuming $M_L = 0, \varepsilon_B = \varepsilon$ and disregarding the concrete and overburden weight, the **actual soil pressure distribution** is



If ε is large enough, a negative soil pressure will result, which is neglected.



The maximum eccentricity without incurring a reduction in footing contact area will be B/6.

Rafts On Clay

The net or ultimate bearing capacity for rafts on clay is found in the same manner as for shallow footings.

The factor of safety produced by a raft construction is:

F should be at least **3** for normal loadings, but it may be reduced to **2** during temporary extreme loading.

Rafts On Clay

If the in the equation ***is zero***, that is

the raft is said to be **fully compensated foundation**. For D_f less than the fully compensated depth, the raft is said to be a ***partially compensated foundation.***

Examples and Problems

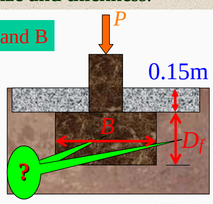
Example 1

An individual square column footing carries an **370kN dead load** and a **335kN live load**. The unconfined compressive strength of the supporting clay is **80kPa**, density is 1840kg/m³. The footing is covered by a 0.15m thick basement slab. Neglect depth correction factors and do not design the structural steel. **Specify the footing size and thickness.**

Solution strategy: Assume D_f and B

Considering soil displacement by footing and slab

Compare actual pressure with the allowable



Solution of Example 1

Bearing Capacity of Clay: $q_a = q_{net}/F = cN_c/F$

From table: N_c=5.7 , shape factor of N_c :1.25

Terzaghi

ϕ	N_c	N_q	N_γ
0.0	5.7	1.0	0.0

Shape factors

B/L	N_c	N_γ
1 (square)	1.25	0.85

Cohesion is estimated: $c = \frac{S_{uc}}{2} = \frac{80\text{kPa}}{2} = 40\text{kPa}$

Using a factor of safety of 3, the allowable pressure:

Total load on the column:

Solution of Example 1 (continued)

Approximate area required:

Try a 2.8m square footing (A=7.84m²), and assuming D_f=0.6m

The actual pressure under the footing:

$$p_{actual} = \frac{705\text{kN}}{7.84\text{m}^2} = 89.9\text{kPa} < q_a \quad [\text{OK}]$$

Concrete density: 2400kg/m3, the pressure surcharge due to 1m² of concrete floor is:

$$p_{slab} = (1\text{m})(1\text{m})(0.15\text{m})\left(2400\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right) = 3.53\text{kPa}$$

Submit 0.15m with 0.6m, the footing self-weight is:

Solution of Example 1 (continued)

Soil surcharge (be taken away) (depth is 0.75m):

$$p_{soil} = (0.75\text{m})\left(1840\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right) = 13.54\text{kPa}$$

Total pressure under the footing:

This is the net actual pressure to be compared to the allowable pressure.

$$p_{net,actual} = 94.02\text{kPa}$$

This is essentially the same as q_a (94.7kPa)

(Solution done)

Example 2

A raft foundation is to be designed for a **36m*60m** building with a total loading of **2.5*10^5kN**. The unconfined compressive strength of the supporting clay is **28.7kPa**, density is 1840kg/m³. The footing is covered by a 0.15m thick basement slab. Neglect depth correction factors. **(a) What should be the raft depth, D_f, for full compensation? (b) What should be the raft depth for a factor of safety of 3?**

Solution of Example 2

For full compensation:

$$D_f = \frac{\dfrac{\text{total load}}{\text{raft area}}}{\rho g} = \frac{\dfrac{(2.5\times10^5\text{kN})(10^3\text{N/kN})}{(36\text{m})(60\text{m})}}{(1840\text{kg/m}^3)(9.81\text{m/s}^2)} = 6.4\text{m}$$

Solution of Example 2

From table: N_c=5.7 , shape factor: B/L=36/60=0.6, use the multiplier of 1.15.

Cohesion is estimated:

$$F = \frac{cN_c}{\dfrac{\text{total load}}{\text{raft area}} - \rho g D_f} \;\Rightarrow\; D_f = \frac{\dfrac{\text{total load}}{\text{raft area}} - \dfrac{cN_c}{F}}{\rho g}$$

$$D_f = \frac{\left(\dfrac{(2.5\times10^5\text{kN})}{(36\text{m})(60\text{m})} - \dfrac{14.35\text{kPa}\times5.7\times1.15}{3}\right)\left(10^3\dfrac{\text{N}}{\text{kN}}\right)}{(1840\text{kg/m}^3)(9.81\text{m/s}^2)} =$$

(Solution done)

Practice Problems

1. A total force of 1600kN is to be supported by a square footing. The footing is to rest directly on sand that has a density of 1900kg/m³ and an angle of internal friction of 38°.

(a) Size of the square footing

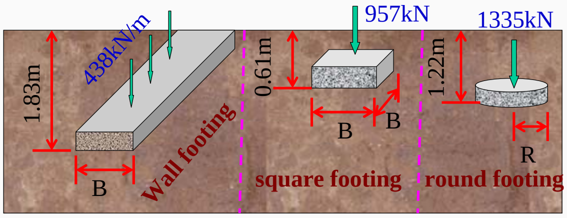
(A) 1.8m (B) 2.4m (C) 2.7m (D) 3.4m

(b) Size of the square footing if it is to be placed 1.5m below the surface.

(A) 1.0m (B) 1.1m (C) 1.3m (D) 1.8m

Practice Problems (continued)

2. The three foundations shown are in a **sandy clay** with the following characteristics: density=1730kg/m³, angle of internal friction= 25° ,and cohesion=19.2kPa. The water table is 10.7m below the ground surface. Use **a factor of safety 2.5** where required.



Practice Problems (continued)

(a) What is the bearing capacity factor for *Nc* according to the Terzaghi and Meyerhof/vesic theory?

(A) 5.7,5.1 (B) 11, 9.7 (C) 25, 21 (D) 350, 270

(b) What is the bearing capacity factor for *Nq* according to the Terzaghi and Meyerhof/vesic theory?

(A) 0, 0 (B) 1.0, 1.0 (C) 3.6, 3.2 (D) 13, 11

(c) What is the bearing capacity factor for *Nr* according to the Terzaghi and Meyerhof/vesic theory?

(A) 0, 0 (B) 1.0, 1.0 (C) 1.9, 1.9 (D) 9.7, 11

(d) What should be the width of the wall footing using the Terzaghi factor? Neglect the weight of the footing.

(A) 0.61m (B) 1.07m (C) 1.16m (D) 1.4m

Practice Problems (continued)

(e) What should be the width of the square footing using the Terzaghi factor? Neglect the weight of the footing.

(A) 0.76m (B) 0.98m (C) 1.46m (D) 1.68m

(f) What should be the radius of the circular footing using the Terzaghi/Vesic factor? Neglect the weight of the footing.

(A) 1.07m (B) 1.16m (C) 1.28m (D) 1.68m

(g) What should be the radius of the circular footing using the Meyerhof factor? Neglect the weight of the footing.

(A) 1.07m (B) 1.13m (C) 1.43m (D) 1.77m

Practice Problems (continued)

(h) What is the allowable bearing capacity of the square footing assuming a width of 1.22m and the water table is at 0.61m? Use the Meyer/Vesic factors.

(A) 254.8kPa (B) 301.7kPa (C) 363.9kPa (D) 526.7kPa

(i) What is the allowable bearing capacity of the square footing assuming a width of 1.22m and the water table is at 0.3m? Use the Meyer/Vesic factors.

(A) 234.6kPa (B) 243.2kPa (C) 296.8kPa (D) 426.1kPa

(j) What is the allowable bearing capacity of the circular footing assuming a radius of 1.22m and the water table is at the ground surface? Use the Meyer/Vesic factors.

(A) 251.0kPa (B) 272.9kPa (C) 363.9kPa (D) 445.3kPa

February 14, 2011

Soil Mechanics and Foundation Engineering Lectures

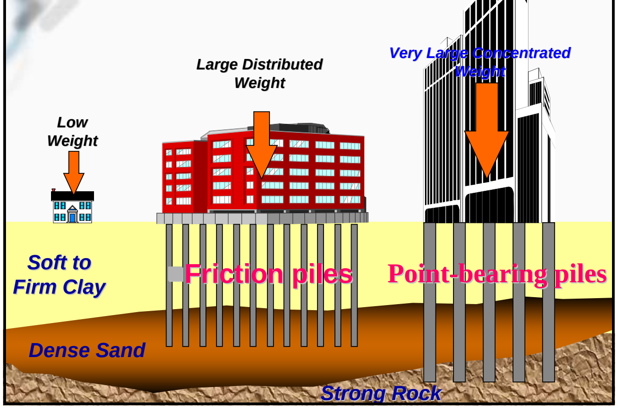
Part 2. Piles and Deep Foundations

Prof. Dr. Songtao XUE

Tohoku Institute of Technology

CONTENT

Piles Foundation Introduction



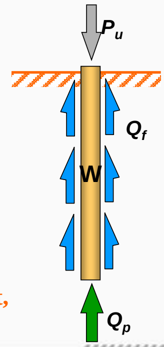
Piles Foundation Introduction

The ultimate static bearing capacity of a single pile is:

The allowable capacity depends on the factor of safety:

F is typically **2 to 3** for both compression and tension piles. The lower of 2 being used when the capacity can be verified by pile loading tests.

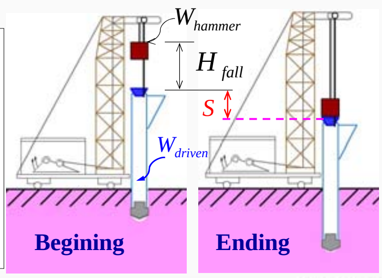
Pile capacities do not consider **settlement,** which might be the controlling factor.



Piles Capacity from Driving Data

- The **safety load** (*safe bearing value*) can be calculated empirically from installation data using the ***Engineer News Record (ENR*** or ***Engineering News)*** equations.

Weight of Hammer:



Piles Capacity from Driving Data

The maximum allowable vertical load, Q_a

➢For drop hammer and single-acting steam hammer when $W_{driven} < W_{hammer}$:

➢Single-acting steam hammer when $W_{driven} > W_{hammer}$:

Substitute the energy ***E*** for ***WH*** for double-acting steam hammer.

Theoretical Point-bearing Capacity

The theoretical point-bearing capacity, also known as the tip resistance or point capacity, of a single pile is:

Meyerhof values of N_q for piles:

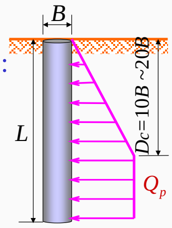
$\phi(^\circ)$	20	25	28	30	32	34	36	38	40	42	45
Driven	8	12	20	25	35	45	60	80	120	160	230
Drilled	4	5	8	12	17	22	30	40	60	80	115

Where ϕ is the internal friction angle

Theoretical Point-bearing Capacity

Case 1: for sands

Cohesionless or granular soils (c=0) :



D_c: critical depth. The tip capacity increase down to D_c , after which it is essentially constant:

Loose sands (relative densities<30%) : D_c=10B
Dense sands (relative densities>70%) : D_c=20B
Interpolated when relative densities between **30%** and **70%**

Theoretical Point-bearing Capacity

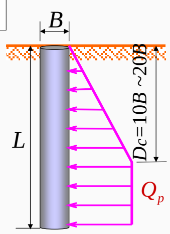
Case 2 cohesive soils $(\phi = 0,\ N_q = 1)$:

- $\rho g D_f$ **is approximately cancelled by the pile weight.**
- **Nc=9 for driven piles of virtually all dimensions.**

Table of N_c for Driven piles: clay

D_f/B	0	1	2	≥4
N_c	6.3	7.8	8.5	9

N_c generally turns to be a value 9 because long length of the pile.



Theoretical Skin-friction Capacity

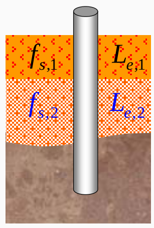
The **skin-friction capacity** (also known as ***side resistance***, ***skin resistance*** and ***shaft capacity***), Q_f :

For piles passing through more than one layers:

$$Q_f = p\sum f_{s,i} L_{e,i}$$

➢Parameters of the equation:

p: the perimeter of the pile.
L_e: effective pile length, can be estimated as the pile length, L, less the seasonal variation, SV, if any.



Parameters of the equation:

$$Q_f = A_s f_s = pf_s L_e = pf_s(L - SV)$$

f_s: **the skin friction coefficient**(unit shaft friction or side friction factor), given as:

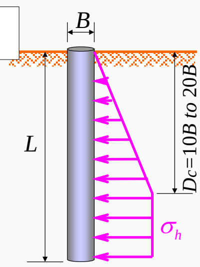
c_A: adhesion, should be obtained from the test, or estimate it from the undrained shear strength, c, as:
- rough concrete, rusty steel, corrugated metal: c_A=c
- **Smooth concrete: 0.8c ≤ c_A ≤c**
- **Clean steel:0.5c≤ c_A ≤0.9c**

Parameters of the equation:

$$f_s = c_A + \sigma_h \tan\delta$$

σ_h:**lateral earth pressure**, depends on the depth, down to a critical depth, D_c, after which it is constant.

k_s : coefficient of lateral earth pressure:
✓Driven piles in clay and silt: $k_{s,dv}$ =1
✓Driven piles in sand:1≤ $k_{s,dv}$ ≤2
The more dense sands, the higher $k_{s,dv}$
✓Drilled plies: $k_{s,dl}$ =0.5$k_{s,dv}$
✓Jetted piles: $k_{s,jt}$ =0.25$k_{s,dv}$



Parameters of the equation necessary for obtain the skin-friction:

$$\sigma_h = k_s(\rho g D - \mu)$$

μ : **pore pressure**, is the hydrostatic pressure at the depth h below the water table:

Another method to determine: f_s

The α-*method* determine the skin friction factor:

α **:adhesion factor**

Typical values of the adhesion factor α :

Cohesion, c	α	
GPa	Range	average
24	--	1.0
48	0.56-0.96	0.83
96	0.34-0.83	0.56
144	0.26-0.78	0.43

Theoretical Skin-friction Capacity

➢Skin-friction capacity for piles through layers of cohesionless sand: c =c_A= 0 ($\sigma' = \rho$ gD- μ)

➢Skin-friction capacity for piles in clay: $k_s = 1$ $\tan\delta = 0,\ f_s = c_A$

➢With β-method for **cohesive clay**, for a pile through one layer: $Q_f = p\beta\sigma' L$

Value of β:

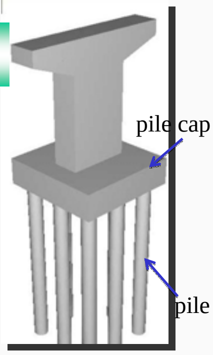
L(m)	0	7.5	15	23	30	38	45	53	60
β	0.3	0.3	0.3	0.27	0.23	0.20	0.18	0.17	0.16

Pile Groups

The capacity of a pile group:

- **For cohesionless soils (sand): the sum of the individual capacities**
- For cohesive soils is the smaller of :
  (a) the sum of the individual capacities
  (b) the capacity assuming block action

The block action capacity is calculated assuming that the piles form a large pier whose dimension are the group's perimeter.

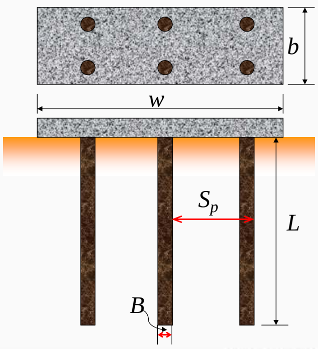


Pile Groups (equations)

Equations for the capacity of a pile group:

c_1 **:average undrained shear strength among the depth of the piles**

c_2 **: undrained shear strength at the pile tips**



Pile Groups (equations)

- **The pile group efficiency is:**

For pile group in clay:

For pile group in sand, efficiencies are similar.

Examples

- A round concrete pile with a diameter of 0.28m is driven 18.3m into wetlands clay. The clay's shear strength and specific weight are 67.03kPa and 1922.2kg/m^3, respectively. At high tide, the water table extends to the ground surface. What is the allowable bearing capacity of the pile?

Solution

The pile's end and surface area:

Examples (solution)

The tip capacity is given for a driven pile N_c=9

For smooth concrete, c_A = 0.9 c *(what we generally assumed)*, the friction capacity is given:

The total capacity is:

$Q_{ult} = \quad = 37.16\text{kN} + 971.26\text{kN} = 1008.42\text{kN}$

With a factor of safety of *F*=3, the allowable load is:

$= 1008.42\text{kN}/3 = 336.14\text{kN}$

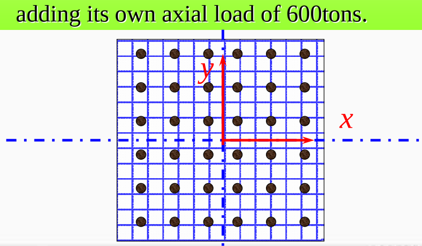
Solution Done

Practice Problems (1)

- A 0.273m diameter steel pile is driven 19.81m into stiff, insensitive clay. The clay has a undrained shear strength of 62.24kPa and its specific weight is 1842kg/m^3. The water table is at the ground surface. The entire pile length is effective.
- (a) what is the end-bearing capacity?
- (b) what is the friction capacity?
- (c) what is the allowable bearing capacity?

Practice Problems (2)

- Tests have shown that a single pile would have, by itself, an uplift capacity of 150tons and a compressive capacity of 500tons. 36 piles are installed on a grid with a spacing of 1.07m. The pile group is capped and connected at the top by a thick concrete slab adding its own axial load of 600tons.



Practice Problems (2)

- (a) Find the maximum moment that the pile group can take in the *x*- and *y*-directions.
- (b) If the concrete slab is sawed completely through along the *y*-axis into tow separate pieces and then reconnected by a flat steel plate at the slab top to prevent drifting, what will be the maximum moment that the pile group can take about the *y*-axis?

Soil Mechanics and Foundation Engineering Lectures

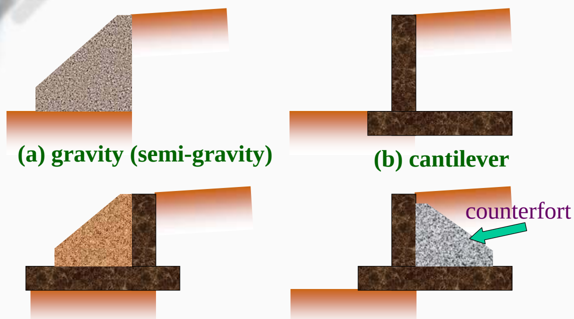
Part 3: Rigid Retaining Wall

Prof. Dr. Songtao XUE

Tohoku Institute of Technology

CONTENT

Type of Retaining Wall



(a) gravity (semi-gravity)

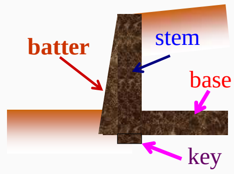
(b) cantilever

(c) buttress

(d) counterfort

Type of Retaining Wall : Cantilever

A **cantilever retaining wall consists** of a **base**, a **stem** and an **optional key**.



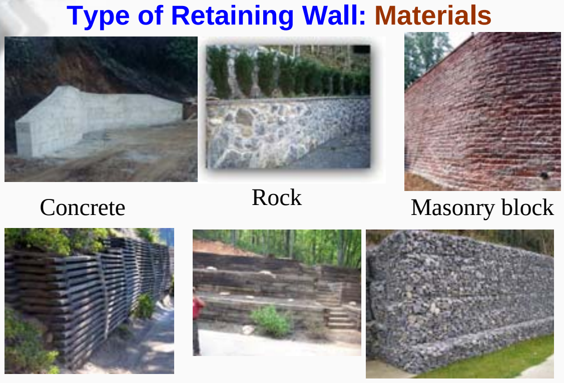
The stem may have a **constant thickness**, or **be tapered**.

The taper is known as **batter**, which is used to:

- "disguise" bending (deflection)
- reduces the quantity of material needed at the top

Cantilever wall are generally intended to be permanent and are made of cast-in-place poured concrete.

Type of Retaining Wall: Materials

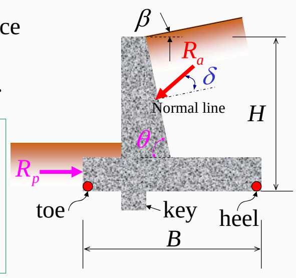


Concrete · Rock · Masonry block

Heavy lumber · Railroad ties · **Gabion wall**

Earth pressure

- ***Earth Pressure*** is the force per unit area exerted by soil on the retaining wall.



- ***Active Earth Pressure*** is

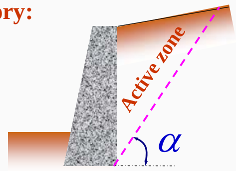
- ***Passive Earth Pressure***

Three common earth pressure theories

- **1, Rankine earth pressure theory:**

Assumes failure occurs along a flat plane behind the wall inclined a angle:

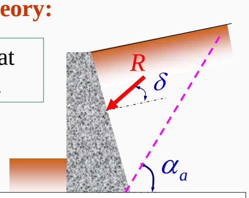
Disregards friction between the wall and the soil.



- **2, Coulomb earth pressure theory:**

Assumes failure occurs along a flat plane and ***include*** wall friction.

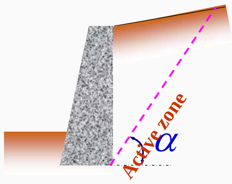
The angle of the failure depends on both ϕ and δ :



When friction is significant, the Coulomb theory can predict lower active pressure than the Rankine theory.

determined by the material type of the cantilever and soil, in case of concrete and coarse sand δ =29-31

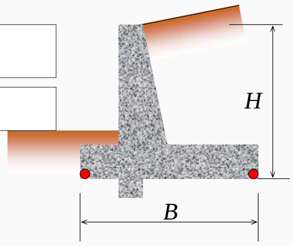
- **3, Log-spiral theory:**



Assumes failure surface will be curved. **Notice that**: the sophistication of a log-spiral solution is probably warranted only for the passive case when where δ is the angle of external friction $\delta \geq 15^{\circ}$

Vertical Soil Pressure

- **Vertical soil pressure** is caused by the soil's own weight and is calculated in the same manner as for a fluid column:



H is measured from the bottom of the base.

Active Earth Pressure

- **Horizontal active earth pressure with level backfill for all soil types is:**

(1)

General form of active (most used):

$$k_a = \frac{\sin^2(\theta+\phi)}{\sin^2\theta \sin(\theta-\phi)\left(1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\beta)}{\sin(\theta-\delta)\sin(\theta+\beta)}}\right)^2}$$

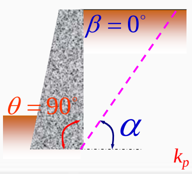
[Coulomb]

Active Earth Pressure (continued 1)

General form of active Rankine equation (can be derived by setting δ =0 in Coulomb equation):

[Rankine]

If the backfill is horizontal and the wall face is vertical



$$\tan^2\left(45^{\circ} - \frac{\phi}{2}\right) = \frac{1-\sin\phi}{1+\sin\phi}$$

[Rankine : horizontal backfill : verticalface]

k_p is the **coefficient of passive earth pressure**

Active Earth Pressure (continued 2)

Active earth pressure equation of

For saturated clays: $\phi = 0^\circ$ and **tension cracks** do not develop near the top of the retaining wall: $k_a = 1$

$p_a = p_v k_a - 2c\sqrt{k_a}$ ⇒ [ ] $[\phi = 0]$

Active earth pressure equation of **granular soil (sand)** :

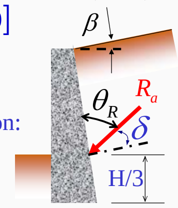
$c = 0$ ⇒ [ ] $[c = 0]$

Total active resultants of **granular soil (sand)** : [ ]

Located at H/3 above the base, direction:

$\theta_R = 90$ [Rankine]

$\theta_R = 90 - \delta$ [Coulomb]



Passive Earth Pressure

- **Equation for horizontal passive earth pressure with level backfill for all soil types is:**

[ ] (2)

General form of passive [ ] equation:

$$k_p = \frac{\sin^2(\theta - \phi)}{\sin^2\theta \sin(\theta + \delta)\left(1 - \sqrt{\frac{\sin(\phi+\delta)\sin(\phi+\beta)}{\sin(\theta+\delta)\sin(\theta+\beta)}}\right)^2}$$

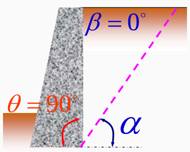
[Coulomb]

Passive Earth Pressure (continued 1)

General form of passive Rankine equation (can be derived by setting δ =0 in Coulomb equation):

$$k_p = \cos\beta \left(\frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}\right) \quad [\text{Rankine}]$$

If the backfill is horizontal and the wall face is vertical:



$$[\ ] = \tan^2\left(45^\circ + \frac{\phi}{2}\right) = \frac{1 + \sin\phi}{1 - \sin\phi}$$

[Rankine : horizontal backfill : verticalface]

Passive Earth Pressure (continued 2)

Passive earth pressure equation of [ ]

For saturated clays: $\phi = 0^\circ$, $k_a = 1$

$p_p = p_v k_p + 2c\sqrt{k_p}$ ⇒ [ ] $[\phi = 0]$

Passive earth pressure equation of [ ]:

$c = 0$ ⇒ [ ] $[c = 0]$

Total active resultants of granular soil :

[ ]

When a wall is first backfilled, or when the toe is excavated, passive pressure may be absent.

Surcharge Loading

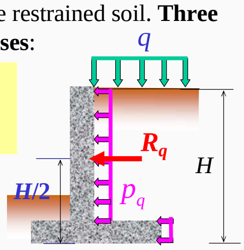
- A **surcharge** is an additional force applied at the exposed upper surface of the restrained soil. **Three kinds of surcharge load cases**:

  - **Uniform load surcharge**
  - **Point load surcharge**
  - **Line load surcharge**

Uniform load surcharge

[ ]

[ ]



R_q acts horizontally at H/2 above the base.

Surcharge Loading: Point load case

If a vertical point load surcharge (e.g., a truck wheel) is applied a distance x back from the wall face:

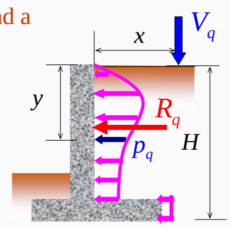
Assume elastic soil performance and a Poisson's ratio of 0.5.

$$p_q = \frac{1.77 V_q m^2 n^2}{H^2 (m^2 + n^2)^3}$$

$$m = \frac{x}{H}, \quad n = \frac{y}{H}$$

$m = 0.4 \ \ if \ \ m < 0.4$

For m=0.4, 0.5, 0.6:



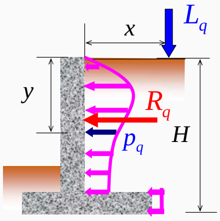
Surcharge Loading: Line load case

For a line load surcharge, L_q, the distribution of pressure behind the wall is given as follows:

$$p_q = \frac{4L_q m^2 n}{\pi H \left(m^2+n^2\right)^2} \quad [m > 0.4]$$

$$R_q = \frac{0.64 L_q}{m^2+1}$$

$$p_q = \frac{0.203 L_q n}{H\left(0.16+n^2\right)^2} \quad [m \le 0.4]$$



$$m = \frac{x}{H}, \quad n = \frac{y}{H}$$

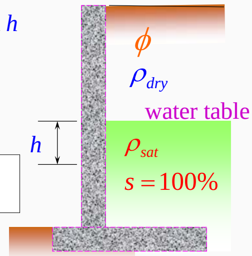
Effective Stress

With submerged construction or when drains become plugged, a water table can exist behind the wall.

The pore pressure (i.e., the *hydrostatic pressure*) at a point h below the water table is:

The saturated soil density:

n : the porosity; e: void ratio.



Effective Stress (continued)

- The **effective pressure** (generally referred to as the *effective stress*) is the difference between the total pressure and the pore pressure.

The total horizontal pressure form the submerged sand is

$$p_h = g\rho_w h + k_a g\left(\rho_{sat}H - \rho_w h\right) = g\left(k_a \rho_{sat} H + \left(1-k_a\right)\rho_w h\right)$$

The increase in horizontal pressure above the saturated condition is the **equivalent hydrostatic pressure** (*equivalent fluid pressure*) caused by the ***equivalent fluid weight***:

Cantilever Retaining Walls: Analysis

- **Targets of analysis:**

✓Have sufficient resistance against overturning and sliding

✓Have adequate structural strength against bending outward

✓The maximum soil pressure under the base must be less than the allowable soil pressure.

- **Nine steps to analyze a retaining wall:**
  - ➢**Step 1:**

Cantilever Retaining Walls: Analysis step

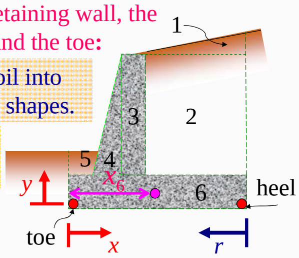
➢**Step 2:** disregarding restraint from passive distribution.
(If it is to be considered, determine the passive earth pressure.)

➢**Step 3:** Find all of the vertical forces acting at the base.

Including: weights of the retaining wall, the soil directly above the hell and the toe:

Dividing the concrete and soil into areas with simple geometric shapes.

Find the centroid of each shape and its moment arm:



Cantilever Retaining Walls: Analysis step

➢**Step 4:** Find the moment about the toe from the forces found in steps 1 though 3.

$$M_{toe} = \sum W_i x_i - R_{a,h} y_a + R_{a,v} x_a$$

h: horizontal,
v: vertical

➢**Step 5:** Determine the location, x_R, and eccentricity ε of the vertical force component:

➢**Step 6:** Check the safety factor, F, against overturning:

$$F_{OT} = \frac{M_{resisting}}{M_{overturning}} = \frac{\sum W_i x_i + R_{a,v} x_{a,v}}{R_{a,h} y_{a,h}}$$

granular soils : $F \ge 1.5$ **cohesive soils :** $F \ge 2$

Cantilever Retaining Walls: Analysis step

➢**Step 7:** The maximum pressure (at the toe) should not exceed the allowable pressure:

➢**Step 8:** Calculate the resistant against sliding.

The active pressure is resisted by: friction and adhesion between the base and the soil, and in the case of a keyed base, also by the shear strength of the soil.

When the base has a key, and has the compressed soil in front of the key

Cantilever Retaining Walls: Analysis step

➢**Step 8:** **(to be continued)**

For a keyless base and for tensioned soil behind the key,

c_A :the adhesion, is zero for granular soil

$\tan\delta$:coefficient of friction is approximately 0.45 for and with silt, 0.35 for silt, and 0.3 for clay

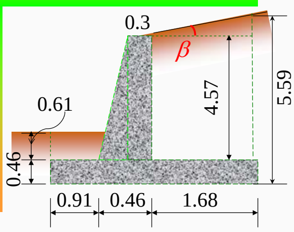
➢**Step 9:** Calculate the factor of safety against sliding:

A lower factor of safety 1.5 is permitted when the passive resultant is disregarded. If the passive resultant is included , the factor of safety should be higher. If the factor is too low, the base length (*B*) can be increased, or a vertical key can be used.

Examples and Problems

EXAMPLE: The unkeyed retaining wall shown has been designed for a backfill of coarse-grained sand with silt having a density of 2000kg/m3. The angle of internal friction, $\phi = 30^\circ$, and the angle of external friction $\delta = 17^\circ$.

The backfill is sloped as shown. The maximum allowable soil pressure is 143.5 kPa. The adhesion is 45.5 kPa. Check the factor of safety against sliding.



Examples and Problems: (solution, Appendix 37.A necessary)

● ***SOLUTION***

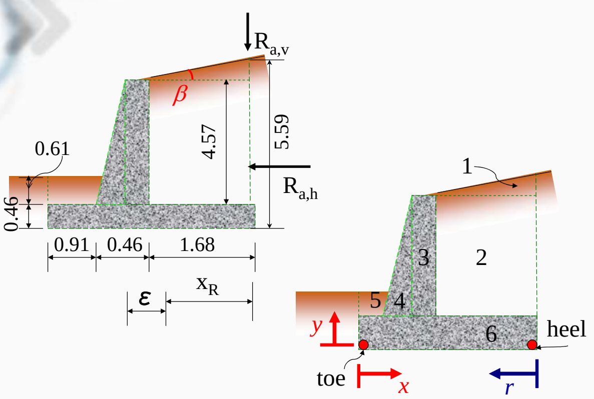
Step 1: $\beta = \arctan\left(\frac{0.56\text{m}}{1.68\text{m}}\right) = \arctan\left(\frac{1}{3}\right) = 18.4^\circ$

From the appendix 37.A $k_v = 1.57\text{kPa}, k_h = 6.28\text{kPa}$

The active earth pressure resultants:

$R_{a,h}$ located 5.59/3=1.86m above the bottom of the base.

Step 2: disregard the passive earth pressure.



Examples and Problems: (solution)

Step 3: **calculate the weights of soil and concrete**

			Per meter of wall		
i	area	ρ	W_i	x_i	M_i
	m²	kg/m³	kN	m	kN·m
1	(0.5)(1.68)(0.56)=0.47	2002	9.22	2.49	22.96
2	(1.68)(4.57)=7.67	2002	150.49	2.21	332.57
3	(0.3)(4.57)=1.37	2402	32.26	1.22	39.35
4	(0.5)(0.15)(4.57)=0.34	2402	8.00	1.01	8.09
5	(0.61)(0.91)=0.56	2002	10.99	0.46	5.05
6	(0.46)(3.05)=1.4	2402	32.96	1.52	50.10
		totals	243.92		458.13

Examples and Problems: (solution)

Step 4: find the moment about the toe:

$$= 458.13\text{kN}\cdot\text{m} - (98.12\text{kN})(1.86\text{m}) + (24.53\text{kN})(3.05\text{m}) = 350.44\text{kN}\cdot\text{m}$$

Step 5: Location and eccentricity of the vertical force:

$$= \frac{350.44\text{kN}\cdot\text{m}}{243.92\text{kN} + 24.53\text{kN}} = 1.3\text{m}$$

$$= 3.05\text{m}/2 - 1.3\text{m} = 0.22\text{m}$$

Step 6: this step skipped

Examples and Problems: (solution)

Step 7: the max pressure at the toe:

$$= \left(\frac{243.92\text{kN} + 24.53\text{kN}}{3.05\text{m}}\right)\left(1 + 6\left(\frac{0.22\text{m}}{3.05\text{m}}\right)\right)$$

$$= 126.1\text{kPa} \le 143.5\text{kPa}, \ [\text{OK}]$$

Solution Done

Examples and Problems: (solution)

Step 8: without KEY, the resistance against the sliding is

$$R_{SL} = \left(\sum W_i + R_{a,v}\right)\tan\delta + c_A B$$
$$= (243.92\text{kN} + 24.53\text{kN})\tan(17^\circ) + (45.5\text{kPa})(3.05\text{m})$$
$$= 220.85\text{kN}$$

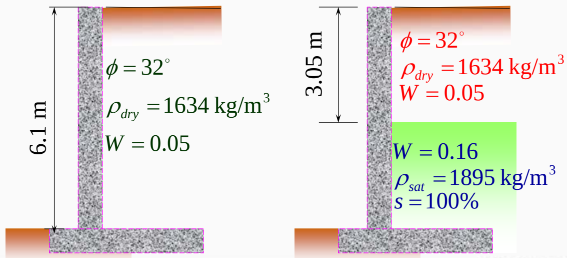
Step 9: and the safety factor against the sliding is

$$F_{SL} = \square = \frac{220.84\text{kN}}{98.12\text{kN}} = 2.25 > 1.5 \quad [\text{OK}]$$

Solution Done

Examples and Problems: (Problems)

Problems 1 A retaining wall is designed for free-draining granular backfill. After several years of operation, the weep-hole become plugged and the water table rises to within 3.05m of the top of the wall.



Examples and Problems: (Problems)

Questions of Problems 1

- **(a) What is the resultant force for the drained case?**
- **(b) What is the point of application for drained case?**
- **(c) What is the resultant force for the plugged case?**
- **(d) What is the point of application of the resultant force for the plugged case?**

Examples and Problems: (Problems)

Problems 2 A 7.92m high retaining wall holds back sand with a 1538kg/m³ drained specific weight. The water table is 3.05m below the top of the wall. The saturated specific weight is 1938kg/m³. the angle of internal friction is 36º.

- (a) What is the active earth resultant?
- (b) What is the location of the active earth pressure resultant?
- (c) if the water table elevation could be reduced 4.88m to the bottom of the wall, what would be the reduction in overturning moment?

Examples and Problems: (Problems)

- **Problems 3** A reinforced concrete retaining wall is used to support a 4.27 m cut in sandy soil. The backfill is level, but a surcharge of 24kN/m^2 is present for a considerable distance behind the wall. Factors of safety of 1.5 against sliding and overturning are required. Customary and reasonable assumptions regarding the proportions can be made. Passive pressure is to be disregarded. The need for a key must be established.

Soil drained specific weight: 2082kg/m^3
Angle of internal friction: 35
Coefficient of friction against concrete: 0.5
Allowable soil pressure: 215.5kPa
Frost line: 1.22m below grade

Examples and Problems: (Problems)

(a) What is the approximate minimum stem height?
(A) 4.27m (B) 4.88 m (C) 5.49 m (D) 6.10m

(b) The surcharge is equivalent to what thickness of backfill soil?
(A) 0.61m (B) 0.91m (C) 1.18m (D) 1.52m

(c) What is the horizontal reaction due to the surcharge? (unit:kN/m)
(A) 35.02 (B) 39.04 (C) 51.08 (D) 56.91

(d) What is the active soil resultant? (unit: kN/m)
(A) 83.18 (B) 99.93 (C) 109.45 (D) 121.12

Examples and Problems: (Problems)

(e) What is the total moment resisting overturning, taken about the heel, per meter of wall? (unit: kN·m)
(A) 533.78 (B) 622.75 (C) 711.71 (D) 854.91

(f) What is the maximum vertical pressure at the toe? (unit: kPa)
(A) 192.92 (B) 215.46 (C) 239.40 (D) 263.34

(g) What is the minimum vertical pressure at the heel? (unit: kPa)
(A) 30.53 (B) 62.24 (C) 114.91 (D) 129.28

Examples and Problems: (Problems)

(h) What is the factor of safety against sliding without a key?
(A) 1.3 (B) 1.4 (C) 1.6 (D) 1.8

(i) What is the factor of safety if a key, 0.53m wide and 0.3m deep, is used?
(A) 1.4 (B) 1.5 (C) 1.7 (D) 2.1

(j) What is the factor of safety against overturning?
(A) 1.5 (B) 1.8 (C) 2.3 (D) 2.6

THE END